Differential Cryptanalysis of T-Function Based Stream Cipher TSC-4

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Abstract. TSC-4 is a T-function based stream cipher with 80-bit key, and proposed as a candidate for ECRYPT eStream project. In this paper, we introduce a differential method to analyze TSC-4. Our attack is based on the vulnerable differential characteristics in the state initialization of TSC-4, and for the chosen IV pairs, the differential probability is up to $2^{-15.40}$ in the case of weak keys. We show that there are about 2^{72} weak keys among the total 2^{80} keys. To recover 8 bits of a weak key needs about $2^{40.53}$ chosen IV pairs. After that, we can search the other 72 key bits by an exhaustive attack.

Keywords: Differential cryptanalysis, T-function, stream cipher, chosen IV attack, TSC-4.

1 Introduction

For a long time, a classical approach for designing stream ciphers is to utilize the Linear Feedback Shift Registers (LFSR) combined with nonlinear Boolean functions. Recently, a new primitive has been introduced to replace the LFSR, which is the T-function (Triangular-function) proposed by Klimov and Shamir [4,5,6]. T-function is a nonlinear function which has some nice properties, such as the operations both available on processors and software implementations, and one single cycle by the appropriate choice, etc. There are some interesting researches on this area in recent years.

At FSE 2005, Hong *et al.* proposed a new class of single T-functions with S-box application [1], and described two stream ciphers TSC-1 and TSC-2. At Asiacrypt 2004, Mitra and Sarkar described a time-memory trade-off attack [8] to break some stream ciphers proposed by Klimov and Shamir. Recently, Kunzli, Junod and Meier proposed the distinguishing attacks [7] applicable to several T-function based stream ciphers (TSC). Based on the above researches, Hong *et al.* proposed a new TSC version called TSC-3 [2], as a candidate to eStream project.

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In 2005 and 2006, Muller and Peyrin [10,11] introduced a linear cryptanalysis attack on the TSC family TSC-1, TSC-2, TSC-3 together with Klimov and Shamir's ciphers. This linear correlation attack can recover the full secret keys of these stream ciphers. Soon, Dukjae Moon *et al.* twisted TSC-3 and proposed the latest version TSC-4 [9] to prevent the distinguishing attack, the time-memory trade-off attack and the linear correlation attack described in above literatures.

At Indocrypt 2006, Fischer, Meier and Berbain *et al.* [3] presented a non-randomness behavior of the full eight-round state initialization for TSC-4. The non-randomness can be detected in the initial state with about 1000 inputs, however, no bias in the keystream of TSC-4 resulting from this non-randomness has been detected yet. So, no substantial attack has been found from the keystream until now.

In this paper, we present an efficient chosen IV differential attack on TSC-4 with weak keys based on two special differential characteristics in the state initialization. The outline of our attack is as follows. Firstly, construct two special differential characteristics in the state initialization of TSC-4. Secondly, investigate what keys and IVs can result in the high occurrence probability of two differential characteristics, and find out the serious non-randomness behavior of the initial state under these weak keys and chosen IVs. Finally, utilizing $2^{40.53}$ chosen IV pairs, we can identify a weak key and recover 8 bits of the key, and recover the full 80-bit weak key by searching exhaustively the other 72 key bits.

This paper is organized as follows. In Section 2, a brief description of TSC-4 is given. Section 3 introduces two special differential characteristics according to the structure of TSC-4. Section 4 presents the chosen IV differential attack on TSC-4. Finally, we conclude the paper in Section 5.

2 A Brief Description of TSC-4

TSC-4 is a synchronous stream cipher optimized for constrained hardware, including the cipher body and the state initialization. The structure of TSC-4 is illustrated in Fig. 1. Its internal state consists of two states X and Y of 4×32 bits each, denoted $X = (x_3, x_2, x_1, x_0)^T$ and $Y = (y_3, y_2, y_1, y_0)^T$, where x_k and y_k ($k = 0, \dots, 3$) are 32-bit words. Let $[x]_i$ denote the i -th least significant bit of a 32-bit word x , then the i -th bit-slice of state X and the i -th bit-slice of Y are defined as

$$[X]_i = \sum_{k=0}^3 [x_k]_i 2^k, \quad [Y]_i = \sum_{k=0}^3 [y_k]_i 2^k, \quad i = 0, \dots, 31. \quad (1)$$

2.1 Cipher Body

We first describe two functions of X and Y . In the case of state X , a 32-bit parameter $\alpha_1(X)$ is computed as a function of X . It is defined by $\alpha_1(X) = (p + c) \oplus p \oplus 2s$ with $p = x_0 \wedge x_1 \wedge x_2 \wedge x_3$ and $s = x_0 + x_1 + x_2 + x_3$ and constant $c_X = 0x51291089$. Here $\wedge$, $\oplus$ and $+$ denote bitwise AND, bitwise XOR

Fig. 1. The structure of TSC-4

and addition operation modulo 2^{32} respectively. The 32-bit parameter $\alpha_2(Y)$ is defined similarly to $\alpha_1(X)$ except for the constant $c_Y=0x12910895$.

In the cipher body, bit-slices $[X]_i$ and $[Y]_i$ are always mapped by two single-cycle S -boxes. The S -boxes are defined as

$$S[16] = \{9, 2, 11, 15, 3, 0, 14, 4, 10, 13, 12, 5, 6, 8, 7, 1\},$$

$$S^6[16] = \{6, 13, 8, 0, 5, 12, 1, 11, 4, 14, 3, 10, 15, 7, 2, 9\}.$$

Now T-functions T_1 and T_2 on input states X and Y are defined as

$$[T_1(X)]_i = \begin{cases} S([X]_i) & \text{if } [\alpha_1(X)]_i = 1, \\ S^6([X]_i) & \text{if } [\alpha_1(X)]_i = 0. \end{cases}$$

$$[T_2(Y)]_i = \begin{cases} S([Y]_i) & \text{if } [\alpha_2(Y)]_i = 1, \\ S^6([Y]_i) & \text{if } [\alpha_2(Y)]_i = 0. \end{cases}$$

Where the bit-slices $[T_1(X)]_i$ and $[T_2(Y)]_i$ ($i = 0, 1, \dots, 31$) are regarded as 4-bit integers similar to equations (1). Then $[X]_i$ will be replaced by $[T_1(X)]_i$, and $[Y]_i$ will be replaced by $[T_2(Y)]_i$ respectively, in this way, the states X and Y are updated by T-function T_1 and T_2 .

The non-linear filter in Fig. 1. produces the actual output keystream from the current internal states X and Y . We divide 32-bit word x_k into four bytes

$x_{k,0}, x_{k,1}, x_{k,2}, x_{k,3}$, where $x_{k,0}$ is the least significant byte, and $x_{k,3}$ is the most significant byte. This is similar to $y_{k,l}$ ($0 \leq k, l \leq 3$). Then we compute six 8-bit temporary variables $a_0, \dots, a_5$ as follows:

$$\begin{aligned} a_0 &= x_{3,3} + y_{1,1}, \\ a_1 &= x_{0,3} + y_{2,1}, \\ a_2 &= x_{2,2} + y_{3,2}, \\ a_3 &= x_{1,2} + y_{0,2}, \\ a_4 &= x_{3,1} + y_{2,3}, \\ a_5 &= x_{0,1} + y_{1,3}, \end{aligned} \tag{2}$$

where, $+$ denotes the addition operation modulo 2^8 . Now the 8-bit keystream z is defined as

$$z = a_0 \oplus (a_1) \ggg 5 \oplus (a_2) \ggg 2 \oplus (a_3) \ggg 5 \oplus (a_4) \ggg 6 \oplus (a_5) \ggg 2, \tag{3}$$

where $\ggg$ denotes the right rotation.

2.2 State Initialization

We now describe how the states X and Y are initialized from a given key and IV.

Key/IV Loading: Let $K = (k_{79}, k_{78}, \dots, k_1, k_0)$ and $IV = (iv_{79}, iv_{78}, \dots, iv_1, iv_0)$ be an 80-bit key and 80-bit IV respectively. Two 128-bit internal states X and Y are initialized as follows:

$$\begin{aligned} X &= \begin{pmatrix} x_3 \\ x_2 \\ x_1 \\ x_0 \end{pmatrix} = \begin{pmatrix} iv_{63} & iv_{62} & \dots & iv_{33} & iv_{32} \\ iv_{31} & iv_{30} & \dots & iv_1 & iv_0 \\ k_{63} & k_{62} & \dots & k_{33} & k_{32} \\ k_{31} & k_{30} & \dots & k_1 & k_0 \end{pmatrix}, \\ Y &= \begin{pmatrix} y_3 \\ y_2 \\ y_1 \\ y_0 \end{pmatrix} = \begin{pmatrix} k_{47} & k_{46} & \dots & \dots & k_{17} & k_{16} \\ k_{15} & \dots & k_0 & k_{79} & \dots & k_{64} \\ iv_{47} & iv_{46} & \dots & \dots & iv_{17} & iv_{16} \\ iv_{15} & \dots & iv_0 & iv_{79} & \dots & iv_{64} \end{pmatrix}. \end{aligned}$$

Warm-up: Once the internal states are initialized, the K and IV are mixed by the following process,

- Step 1, run cipher body once to produce a single 8-bit output,
- Step 2, rotate x_1 and y_0 to the left by 8 bits,
- Step 3, XOR the output z to the least significant 8 bits of x_1 and y_0 .

The state initialization is completely finished by repeating the above three steps 8 times.

3 Two Special Differential Characteristics in the State Initialization

In this section, we will construct two special differential characteristics which will occur with obvious probability advantage for a large amount of keys.

Suppose that IV and IV' are two different IVs, and we denote them as an IV pair (IV, IV') . Let $X = (x_3, x_2, x_1, x_0)^T$ and $Y = (y_3, y_2, y_1, y_0)^T$ are two 128-bit initial states corresponding to IV . $X' = (x'_3, x'_2, x'_1, x'_0)^T$ and $Y' = (y'_3, y'_2, y'_1, y'_0)^T$ are states related to IV' . Denote the differential $X \oplus X'$ as $\Delta X = (\Delta x_3, \Delta x_2, \Delta x_1, \Delta x_0)^T$, and $Y \oplus Y'$ as $\Delta Y = (\Delta y_3, \Delta y_2, \Delta y_1, \Delta y_0)^T$, where $\Delta x_k = x_k \oplus x'_k$, $\Delta y_k = y_k \oplus y'_k$, $k = 0, \dots, 3$.

All the 16 bytes of ΔY can be represented as a matrix $\Delta Y = (\Delta y_{k,l})_{4 \times 4}$ with $\Delta y_{k,l} = y_{k,l} \oplus y'_{k,l}$, where $0 \leq k, l \leq 3$.

$$\Delta Y = \begin{pmatrix} \Delta y_3 \\ \Delta y_2 \\ \Delta y_1 \\ \Delta y_0 \end{pmatrix} = \begin{pmatrix} \Delta y_{3,3} & \Delta y_{3,2} & \Delta y_{3,1} & \Delta y_{3,0} \\ \Delta y_{2,3} & \Delta y_{2,2} & \Delta y_{2,1} & \Delta y_{2,0} \\ \Delta y_{1,3} & \Delta y_{1,2} & \Delta y_{1,1} & \Delta y_{1,0} \\ \Delta y_{0,3} & \Delta y_{0,2} & \Delta y_{0,1} & \Delta y_{0,0} \end{pmatrix}.$$

In the Key/IV loading process, it is obvious that the IV bits $iv_{48}, \dots, iv_{63}$ (to initial state X) and $iv_{64}, \dots, iv_{79}$ (to initial state Y) occur only one time respectively, and the other IV bits occur twice. This means that one nonzero bit difference in $\{iv_{64}, iv_{49}, \dots, iv_{79}\}$ results in only one nonzero difference bit in ΔY .

We denote the IV differential $IV \oplus IV'$ as $\Delta IV = (\Delta iv_{79}, \dots, \Delta iv_0)$ with $\Delta iv_j = iv_j \oplus iv'_j$, where $0 \leq j \leq 79$. Then we announce for the same key K , the chosen IV pair holds that IV and IV' are only different at one bit $\Delta iv_{j_0} = 1$ and the other 79 bits of them are all the same, where $64 \leq j_0 \leq 79$. After the key/IV loading process, we can get the initial state of X and Y are $\Delta X^0 = (0, 0, 0, 0)^T$ and $\Delta Y^0 = (0, 0, 0, \Delta y_0^0)^T$ respectively, where Δy_0^0 only has one nonzero bit difference (i.e., the hamming weight of Δy_0^0 is 1). In this paper, we select $j_0 = 66$ i.e., $\Delta y_0^0 = 0x4$. The choice for the position j_0 depends on the occurrence probability of the expected differential characteristic in the state initialization process, the details are described in Subsection 4.1. The state initialization in Section 2.2 includes 8 warm-up processes, and we denote each warm-up process as one round.

Now we construct two differential characteristics Ω_X and Ω_Y , and discuss the interaction between them.

Differential characteristic Ω_X : Ω_X is an 8-round differential in the state initialization such that the difference of initial state X after Key/IV loading process is $\Delta X^0 = (0, 0, 0, 0)^T$, and the output difference of internal state X after Step 3 at the i -th round is $\Delta X^i = (0, 0, 0, 0)^T$, $i = 1, \dots, 8$, i.e.,

$$\Omega_X : X^0 \xrightarrow[\text{Round 1}]{\text{Step 3}} X^1 \xrightarrow[\text{Round 2}]{\text{Step 3}} X^2 \longrightarrow \dots \xrightarrow[\text{Round 8}]{\text{Step 3}} X^8.$$

Differential characteristic Ω_Y : Ω_Y behaves that, the difference of initial state Y after Key/IV loading process is $\Delta Y^0 = (0, 0, 0, 4)^T$, and the output difference of internal state Y after Step 1 before Step 2 at the i -th round is ΔY^i which holds that 6 differential bytes $\Delta Y_{3,2}^i, \Delta Y_{2,3}^i, \Delta Y_{2,1}^i, \Delta Y_{1,3}^i, \Delta Y_{1,1}^i$ and $\Delta Y_{0,2}^i$ are all 0. Here $i = 1, \dots, 8$.

$$\Omega_Y : Y^0 \xrightarrow[\text{Round 1}]{\text{Step 1}} \Delta Y^1 \xrightarrow[\text{Round 2}]{\text{Step 1}} \Delta Y^2 \longrightarrow \dots \xrightarrow[\text{Round 8}]{\text{Step 1}} \Delta Y^8.$$

The differential characteristic Ω_Y is illustrated in Fig. 2. The blank means that the difference byte is 0.

Fig. 2. The differential characteristic Ω_Y in the state initialization

It is obvious that, if IV and IV' with the same key K are different at only one bit $\Delta iv_{66} = 1$, the initial state differential ΔX^0 is $(0, 0, 0, 0)^T$ and ΔY^0 is $(0, 0, 0, 4)^T$. And then, by the equations (2) and (3), it is easy to confirm that, ΔX^0 and the choice of ΔY^1 guarantee the keystream output differential Δz_r^1 at the first round is 0. So, from Step 3 of the warm-up process in Subsection 2.2, the difference ΔX^1 is always $(0, 0, 0, 0)^T$ after the first round. Similarly, ΔX^1 and the choice of ΔY^2 guarantee ΔX^2 is always $(0, 0, 0, 0)^T$ after the second round, and so on. This means that, under the condition IV and IV' are different at only one bit $\Delta iv_{66} = 1$, if the differential characteristic Ω_Y holds, then the

differential characteristic Ω_X holds with probability 1. And we can only focus on the occurrence probability of Ω_Y in the next sections.

4 Chosen IV Differential Attack on TSC-4

In this section, we introduce a chosen IV differential attack on TSC-4.

4.1 Weak Keys Resulting in the High Occurrence Probability of Differential Characteristic Ω_Y

We now investigate what keys and IVs can result in the high occurrence probability of differential characteristic Ω_Y in Section 3.

Experiment 1. For each j_0 ($j_0 = 64, \dots, 79$), we randomly select 2^{38} (K, IV, IV') triplets where IV and IV' are only different at one bit $\Delta iv_{j_0} = 1$. For each (K, IV, IV') triplet, we perform two state initialization processes with (K, IV) and (K, IV') to search for Ω_Y . We find that there are more Ω_Y holding for $j_0 = 66$ or 68 . Especially, for $j_0 = 66$, the number of obtained Ω_Y is up to 4983. Since each Ω_Y corresponds to one (K, IV) pair, we can further investigate the statistical property of 4983 (K, IV) pairs obtained above. Firstly, we perform one Key/IV loading process with each obtained (K, IV) pair, then 4983 initial state Y^0 can be obtained. Secondly, we employ a χ^2 test for 16 states of each $[Y^0]_i$ ($i = 0, 1, \dots, 31$). Finally, list the χ^2 distributions of $[Y^0]_i$ which are serious unbalance, and the result is illustrated in Table 1.

Table 1. χ^2 distributions and χ^2 values of $[Y^0]_i$ ($i = 0, 1, \dots, 4$)

k	$[Y^0]_0$	$[Y^0]_1$	$[Y^0]_2$	$[Y^0]_3$	$[Y^0]_4$
0	169	335	0	636	58
1	156	50	0	73	0
2	229	57	4	63	98
3	379	103	5	624	0
4	183	71	0	97	4594
5	376	101	0	563	15
6	400	116	0	602	1
7	356	839	0	50	7
8	176	63	0	93	30
9	359	115	0	707	1
10	406	92	0	665	4
11	323	841	0	47	4
12	322	105	12	596	1
13	318	865	9	68	76
14	333	791	2413	64	1
15	498	439	2540	35	93
χ^2	486.2	5201.4	34429.2	4047.9	62874.7

From Table 1, we find that the 5 least significant columns of $[Y^0]_i$ reveal the serious unbalance for χ^2 value which is more than 100, where $\chi^2 = \sum_{k=0}^{15} \frac{(n_k - np_k)^2}{np_k}$, n_k is the frequency of the 4-bit integer $[Y^0]_i$ being k , $n = \sum_{k=0}^{15} n_k$ and $p_k = \frac{1}{16}$. Since K and IV are embedded into Y^0 , we hope to deduce some linear correlations between K and IV by the unbalance of $[Y^0]_i$.

For the initial state $Y^0 = (y_3^0, y_2^0, y_1^0, y_0^0)^T$ initialed by IV and K after the Key/IV loading process, we know that $[y_0^0]_i = iv_{64+i}$, $[y_1^0]_i = iv_{16+i}$, $[y_2^0]_i = k_{64+i}$ and $[y_3^0]_i = k_{16+i}$, where $i = 0, 1, \dots, 4$. So, $[Y^0]_i$ can be represented as

$$[Y^0]_i = iv_{64+i} + iv_{16+i} \cdot 2 + k_{64+i} \cdot 2^2 + k_{16+i} \cdot 2^3, \quad i = 0, 1, \dots, 4. \quad (4)$$

Especially, we can find some strong linear correlations between K and IV . For example, $[Y^0]_4$ concentrates on the point 0x4 with probability $\frac{4594}{4983} = 0.9129$ by the sixth column of Table 1. So from $[Y^0]_4 = 0x4$ and equation (4), we get the following conditional probability

$$\Pr(k_{20} = 0, k_{68} = 1, iv_{20} = 0, iv_{68} = 0 | \Omega_Y) = 0.9129. \quad (5)$$

We try to explore some other more complex linear correlations between K and IV by observing the χ^2 distribution of $[Y^0]_i$. We firstly investigate the possible linear equations from $[Y^0]_0$. In order to find the linear correlations easily with high probability, we list the distribution of $[Y^0]_0$ as a matrix in Fig. 3.

<i>cd</i> \ <i>ab</i>	00	01	10	11
00	169	156	229	379
01	183	376	400	356
10	176	359	406	323
11	322	318	333	498

Fig. 3. 4×4 matrix of $[Y]_0$, ab represents $k_{16}k_{64}$, and cd is $iv_{16}iv_{64}$

From Fig. 3, we know that two linear equations $iv_{64} \oplus iv_{16} = 1$ and $k_{64} \oplus k_{16} = 1$ hold concurrently in the four bold items which correspond to $5 = (0101)_2$, $6 = (0110)_2$, $9 = (1001)_2$ and $10 = (1010)_2$ respectively.

From $\Pr([Y]_0 = 5, 6, 9, 10 | \Omega_Y) = \frac{1541}{4983} = 0.3093$, we get

$$\Pr(iv_{64} \oplus iv_{16} = 1, k_{64} \oplus k_{16} = 1 | \Omega_Y) = 0.3093. \quad (6)$$

Clearly, the probability of (6) is much more than the average value 0.25.

By studying the 3rd, 4-th and 5-th columns of Table 1, we can conclude that

$$\Pr(k_{17} = 1, k_{65} = 1, iv_{65} \oplus iv_{17} = 1 | \Omega_Y) = 0.3323, \quad (7)$$

$$\Pr(k_{18} = 1, k_{66} = 1, iv_{18} = 1 | \Omega_Y) = 0.9939, \quad (8)$$

$$\Pr(k_{19} \oplus k_{67} = 1, iv_{19} \oplus iv_{67} = 1 | \Omega_Y) = 0.5091. \quad (9)$$

Summing up all the linear equations (5)-(9), we define the following sets,

$$\begin{aligned} A_0 &= \{K | k_{64} \oplus k_{16} = 1\}, \\ A_1 &= \{K | k_{17} = 1, k_{65} = 1\}, \\ A_2 &= \{K | k_{18} = 1, k_{66} = 1\}, \\ A_3 &= \{K | k_{19} \oplus k_{67} = 1\}, \\ A_4 &= \{K | k_{20} = 0, k_{68} = 1\}. \end{aligned}$$

$$\begin{aligned} B_0 &= \{IV | iv_{64} \oplus iv_{16} = 1\}, \\ B_1 &= \{IV | iv_{65} \oplus iv_{17} = 1\}, \\ B_2 &= \{IV | iv_{18} = 1\}, \\ B_3 &= \{IV | iv_{19} \oplus iv_{67} = 1\}, \\ B_4 &= \{IV | iv_{20} = 0, iv_{68} = 0\}. \end{aligned}$$

Let

$$\mathcal{A} = \bigcap_{j=0}^4 A_j, \quad \mathcal{B} = \bigcap_{j=0}^4 B_j.$$

We can get the following probability by searching 4983 (K, IV) pairs in Experiment 1.

$$\Pr(K \in \mathcal{A}, IV \in \mathcal{B} | \Omega_Y) = \frac{386}{4983} = 2^{-3.69}. \quad (10)$$

And it is obvious that

$$\Pr(\Omega_Y) = \frac{4983}{2^{38}} = 2^{-25.71}. \quad (11)$$

Because $\Pr(K \in \mathcal{A}, IV \in \mathcal{B}) = 2^{-14}$, by the conditional probability formula with (10) and (11), we get

$$\Pr(\Omega_Y | K \in \mathcal{A}, IV \in \mathcal{B}) = \frac{\Pr(K \in \mathcal{A}, IV \in \mathcal{B} | \Omega_Y) \Pr(\Omega_Y)}{\Pr(K \in \mathcal{A}, IV \in \mathcal{B})} = 2^{-15.40}. \quad (12)$$

We define the key K which falls into set $\mathcal{A}$ as a weak key. For 80 bits key size, the number of weak keys is $2^{80-8} = 2^{72}$. From equation (12), we know that, for a chosen IV pair (IV, IV') with $\{IV, IV'\} \subset \mathcal{B}$, if K is a weak key, Ω_Y occurs with high probability $2^{-15.40}$. We can verify the above probability by the statistical Experiment 2.

Experiment 2. We randomly select 2^{10} weak keys. For each weak key, randomly select 2^{20} (IV, IV') pairs where IV and IV' are only different at one bit $\Delta iv_{66} = 1$, and both belong to set $\mathcal{B}$. Then we get the total occurrence frequency of the differential characteristic Ω_Y is 26086.

Experiment 2 means that $\Pr(\Omega_Y | K \in \mathcal{A}, IV \in \mathcal{B})$ is about $\frac{26086}{2^{10} \times 2^{20}} = 2^{-15.33}$ which confirms our result in (12) again.

4.2 Identifying and Recovering the Weak Keys

From the above subsection, we know that, for a chosen IV pair, if K is a weak key, Ω_Y happens with high probability $2^{-15.40}$. But this is not enough to identify and recover a weak key, we need more information. Denote the first actual keystream output byte which is used to encrypt a plaintext byte as z_s^1 , and z_s^1 is generated by IV and K . Similarly, $z_s'^1$ corresponds to IV' and K . Let Δz_s^1 be $z_s^1 \oplus z_s'^1$, and $[\Delta z_s^1]_k$ be the k -th least significant bit of the differential byte Δz_s^1 , $k = 0, 1, \dots, 7$. Then the bias of $[\Delta z_s^1]_k$ will provide a solution to identify a weak key.

Experiment 3. In Experiment 2, we have obtained 26086 differential characteristics Ω_Y . And we know each Ω_Y corresponds to one (K, IV, IV') triplet, then we utilize each (K, IV, IV') triplet to run cipher body once and produce two first 8-bit keystream outputs z_s^1 and $z_s'^1$. Then 26086 Δz_s^1 bytes can be obtained.

Denote the number of $[\Delta z_s^1]_k$ being '0' as $n_{k,0}$, and the number of $[\Delta z_s^1]_k$ being '1' as $n_{k,1}$. The 0-1 bias of $[\Delta z_s^1]_k$ can be represented as $\varepsilon_k = |\frac{n_{k,0}}{n_{k,0} + n_{k,1}} - \frac{1}{2}|$. From Experiment 3, for a weak key with chosen IV pairs, if the differential characteristic Ω_Y occurs, the bias of $[\Delta z_s^1]_k$ is listed in Table 2.

Table 2. The bias of $[\Delta z_s^1]_k$ with weak keys ($k = 0, 1, \dots, 7$)

k	0	1	2	3	4	5	6	7
$ \log_2 \varepsilon_k $	4.38	3.54	5.09	5.90	4.91	4.81	6.15	5.02

For a weak key with chosen IV pairs, we adopt $[\Delta z_s^1]_1$ which holds with the highest bias, and get the following conclusion by Experiment 3.

$$\Pr([\Delta z_s^1]_1 = 0 | \Omega_Y, K \in \mathcal{A}, IV \in \mathcal{B}) = \frac{1}{2} + 2^{-3.54}. \quad (13)$$

In an actual attack, we can not get Ω_Y directly except for the actual keystream output. Then we need to know the bias of $[\Delta z_s^1]_1$ without the condition Ω_Y . By further experiments, we can find that if Ω_Y does not happen, for any K and IV , the probability $\Pr([\Delta z_s^1]_1 = 0 | \overline{\Omega_Y})$ is very close to $\frac{1}{2}$. Then for a weak key and the chosen IV pairs, from (12) and (13), we can get

$$\Pr([\Delta z_s^1]_1 = 0 | K \in \mathcal{A}, IV \in \mathcal{B}) = \frac{1}{2} + 2^{-15.40} \times 2^{-3.54} = \frac{1}{2} + 2^{-18.94}. \quad (14)$$

In order to improve the correctness for identifying a weak key, we need to explore the 0-1 bias of $[\Delta z_s^1]_k$ for a strong key. A strong key means that the key $K \in \overline{\mathcal{A}}$.

Utilizing 4983 keys and IV pairs obtained by Experiment 1, we get the probability

$$\Pr(K \in \overline{\mathcal{A}}, IV \in \mathcal{B} | \Omega_Y) = \frac{151}{4983} = 2^{-5.04}. \quad (15)$$

Since $\Pr(K \in \overline{\mathcal{A}}, IV \in \mathcal{B}) = (1 - 2^{-8}) \times 2^{-6} = 2^{-6.01}$, from (15), it is easy to deduce the probability

$$\Pr(\Omega_Y | K \in \overline{\mathcal{A}}, IV \in \mathcal{B}) = \frac{\Pr(K \in \overline{\mathcal{A}}, IV \in \mathcal{B} | \Omega_Y) \Pr(\Omega_Y)}{\Pr(K \in \overline{\mathcal{A}}, IV \in \mathcal{B})} = 2^{-24.74}. \quad (16)$$

Experiment 4. We randomly select 2^{10} strong keys. For each strong key, randomly select 2^{24} chosen IV pairs, and we can get that the total frequency that Ω_Y occurs is 645. Therefore $\Pr(\Omega_Y | K \in \overline{\mathcal{A}}, IV \in \mathcal{B})$ is about $\frac{645}{2^{10} \times 2^{24}} = 2^{-24.67}$. It is clear that, Experiment 4 verifies the result in (16).

In Experiment 4, we can also get 645 Δz_s^1 corresponding to 645 Ω_Y . By counting the number of '0' among 645 $[\Delta z_s^1]_1$, we calculate the following probability under a strong key and the chosen IV pairs,

$$\Pr([\Delta z_s^1]_1 = 0 | \Omega_Y, K \in \overline{\mathcal{A}}, IV \in \mathcal{B}) = \frac{1}{2} + 2^{-3.68}. \quad (17)$$

By combination of equations (16) and (17), for a strong key and the chosen IV pairs, similarly to (14), we get the probability

$$\Pr([\Delta z_s^1]_1 = 0 | K \in \overline{\mathcal{A}}, IV \in \mathcal{B}) = \frac{1}{2} + 2^{-24.74} \times 2^{-3.68} = \frac{1}{2} + 2^{-28.42}. \quad (18)$$

We use (14) and (18) to identify the weak keys. Approximate the binomial distribution with the normal distribution. Denote the total number of samples as N , the mean as μ , and the standard variance as σ .

From (14), $p = \frac{1}{2} + 2^{-18.94}$, $\mu = Np$ and $\sigma = \sqrt{Np(1-p)}$.

From (18), $p' = \frac{1}{2} + 2^{-28.42}$, $\mu' = Np'$ and $\sigma' = \sqrt{Np'(1-p')}$.

For the normal distribution, the cumulative function gives value $1 - 2^{-9.53}$ at $3\sigma'$, and value 0.023 at -2σ .

If the following relation holds

$$\mu - \mu' \geq 3\sigma' + 2\sigma, \quad (19)$$

a strong key will be wrongly identified as a weak key (false positive) with probability $2^{-9.53}$, and each weak key is not identified as a weak key (false negative) with probability 0.023. This means that the weak keys can be successfully identified. By solving (19), the amount of IV pairs required is $N = 2^{40.53}$.

Identifying a weak key means that we can recover 8 key bits. The other 72 key bits can be recovered by searching exhaustively.

5 Conclusion

In this paper, we developed a differential attack on TSC-4. Utilizing the structure of T-function in TSC-4, we constructed a special differential characteristic with high probability which incurs about 2^{72} weak keys. $2^{40.53}$ chosen IV pairs can be used to identify a weak key and recover 8 bits of the weak key. We can recover the other 72 key bits by exhaustive search.

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